



Theory of rumour spreading in complex social networks

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Abstract

We introduce a general stochastic model for the spread of rumours, and derive mean-field equations that describe the dynamics of the model on complex social networks (in particular, those mediated by the Internet). We use analytical and numerical solutions of these equations to examine the threshold behaviour and dynamics of the model on several models of such networks: random graphs, uncorrelated scale-free networks and scale-free networks with assortative degree correlations. We show that in both homogeneous networks and random graphs the model exhibits a critical threshold in the rumour spreading rate below which a rumour cannot propagate in the system. In the case of scale-free networks, on the other hand, this threshold becomes vanishingly small in the limit of infinite system size. We find that the initial rate at which a rumour spreads is much higher in scale-free networks than in random graphs, and that the rate at which the spreading proceeds on scale-free networks is further increased when assortative degree correlations are introduced. The impact of degree correlations on the final fraction of nodes that ever hears a rumour, however, depends on the interplay between network topology and the rumour spreading rate. Our results show that scale-free social networks are prone to the spreading of rumours, just as they are to the spreading of infections. They are relevant to the spreading dynamics of chain emails, viral advertising and large-scale information dissemination algorithms on the Internet.

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1. Introduction

Rumours are an important form of social communications, and their spreading plays a significant role in a variety of human affairs. The spread of rumours can shape the public opinion in a country [1], greatly impact financial markets [2,3] and cause panic in a society during wars and epidemics outbreaks. The information content of rumours can range from simple gossip to advanced propaganda and marketing material. Rumour-like mechanisms form the basis for the phenomena of viral marketing, where companies exploit social networks of their customers on the Internet in order to promote their products via the so-called ‘word-of-email’ and ‘word-of-web’ [4]. Finally, rumour-mongering forms the basis for an important class of

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communication protocols, called gossip algorithms, which are used for large-scale information dissemination on the Internet, and in peer-to-peer file sharing applications [5,6].

Rumours can be viewed as an “infection of the mind”, and their spreading shows an interesting resemblance to that of epidemics. However, unlike epidemic spreading quantitative models and investigations of rumour spreading dynamics have been rather limited. A standard model of rumour spreading, was introduced many years ago by Daley and Kendal [7,8]. The Daley–Kendal (DK) model and its variants, such as the Maki–Thompson (MK) model [9], have been used extensively in the past for quantitative studies of rumour spreading [10–13]. In the DK model a closed and homogeneously mixed population is subdivided into three groups, those who are ignorant of the rumour, those who have heard it and actively spread it, and those who have heard the rumour but have ceased to spread it. These groups are called ignorants, spreaders and stiflers, respectively. The rumour is propagated through the population by *pair-wise* contacts between spreaders and others in the population, following the law of mass action. Any spreader involved in a pair-wise meeting attempts to ‘infect’ the other individual with the rumour. In the case this other individual is an ignorant, it becomes a spreader. In the other two cases, either one or both of those involved in the meeting learn that the rumour is ‘known’ and decide not to tell the rumour anymore, thereby turning into stiflers [8]. In the Maki–Thompson variant of the above model the rumour is spread by *directed* contacts of the spreaders with others in the population. Furthermore, when a spreader contacts another spreader only the *initiating* spreader becomes a stifler.

An important shortcoming of the above class of models is that they either do not take into account the topology of the underlying social interaction networks along which rumours spread (by assuming a homogeneously mixing population), or use highly simplified models of the topology [11,12]. While such simple models may adequately describe the spreading process in small-scale social networks, via the word-of-mouth, they become highly inadequate when applied to the spreading of rumours in large social interaction networks, in particular, those which are mediated by the Internet. Such networks, which include email networks [14–16], social networking sites [17] and instant messaging networks [18] typically number in tens of thousands to millions of nodes. The topology of such large social networks shows highly complex connectivity patterns. In particular, they are often characterized by a highly right-skewed degree distribution, implying the presence of a statistically significant number of nodes in these networks with a very large number of social connections [14,15,17,18].

A number of recent studies have shown that introducing the complex topology of the social networks along which a rumour spreads can greatly impact the dynamics of the above models. Zanette performed simulations of the deterministic MK model on both static [19] and dynamic [20] small-world networks. His studies showed that on small-world networks with varying network randomness the model exhibits a critical transition between a regime where the rumour “dies” in a small neighbourhood of its origin, and a regime where it spreads over a finite fraction of the whole population. Moreno et al. studied the stochastic version of the MK model on scale-free networks, by means of Monte Carlo simulations [21], and numerical solution of a set of mean-field equations [22]. These studies revealed a complex interplay between the network topology and the rules of the rumour model and highlighted the great impact of network heterogeneity on the dynamics of rumour spreading. However, the scope of these studies were limited to *uncorrelated* networks. An important characteristic of social networks is the presence of assortative degree correlations, i.e., the degrees of adjacent vertices is positively correlated [14,15,23,24]. Furthermore, the mean-field equations used in Ref. [22] were postulated without a derivation.

In this paper we make several contributions to the study of rumour dynamics on complex social networks. First of all, we introduce a new model of rumour spreading on complex networks which, in comparison with previous models, provides a more realistic description of this process. Our model unifies the MK model of rumour spreading with the susceptible-infected-removed (SIR) model of epidemics, and has both of these models as its limiting cases. Secondly, we describe a formulation of this model on networks in terms of interacting Markov chains (IMC) [25], and use this framework to derive, from first-principles, mean-field equations for the dynamics of rumour spreading on complex networks with arbitrary degree correlations. Finally, we use approximate analytical and exact numerical solutions of these equations to examine both the steady-state and the time-dependent behaviour of the model on several models of social networks: homogeneous networks, Erdős–Rényi

(ER) random graphs, uncorrelated scale-free networks and scale-free networks with assortative degree correlations.

We find that, as a function of the rumour spreading rate, our model shows a new critical behaviour on networks with bounded degree fluctuations, such as random graphs, and that this behaviour is absent in scale-free networks with unbounded fluctuations in node degree distribution. Furthermore, the initial spreading rate of a rumour is much higher on scale-free networks as compared to random graphs. This spreading rate is further increased when assortative degree correlations are introduced. The final fraction of nodes which ever hears a rumour (we call this the final size of a rumour), however, depends on an interplay between the model parameters and degree–degree correlations. Our findings are relevant to a number of rumour-like processes taking place on complex social networks. These include the spreading of chain emails and Internet hoaxes, viral advertising and large-scale data dissemination in computer and communication networks via the so-called gossip protocols [5].

The rest of this paper is organized as follows. In Section 2 we describe our rumour model. In Section 3 a formulation of the model within the framework of IMC is given, and the corresponding mean-field equations are derived. In Section 4 we present analytical results for the steady-state behaviour of our model for both homogeneous and inhomogeneous social networks. This is followed in Section 5 by numerical investigations of the steady state and dynamics of the model on several models of social networks: the ER random graph, the uncorrelated scale-free networks and the assortatively correlated SF networks. We close this paper in Section 6 with conclusions.

2. A general model for rumour dynamics on social networks

The spread of rumours is a complex socio-psychological process. An adequate modelling of this process requires both a correct description of the underlying social networks along which rumours spread and a quantitative formulation of various behavioural mechanisms that motivate individuals to participate in the spread of a rumour. The model described below is an attempt to formalize and simplify these behavioural mechanisms in terms of a set of simple but plausible rules.

Our model is defined in the following way. We consider a population consisting of N individuals which, with respect to the rumour, are subdivided into ignorants, spreaders and stiflers. Following Maki and Thompson [9], we assume that the rumour spreads by *directed* contact of the spreaders with others in the population. However, these contacts can only take place along the links of an undirected social interaction network $G = (V, E)$, where V and E denote the vertices and the edges of the network, respectively. The contacts between the spreaders and the rest of the population are governed by the following set of rules

- Whenever a spreader contacts an ignorant, the ignorant becomes a spreader at a rate λ .
- When a spreader contacts another spreader or a stifier the *initiating* spreader becomes a stifier at a rate α .

In the above, the first rule models the tendency of individuals to accept a rumour only with a certain probability which, loosely speaking, depends on the urgency or credibility of a rumour. The second rule, on the other hand, models the tendency of individuals to lose interest in spreading a rumour when they learn, through contacts with others, that the rumour has become stale news, or is false. In both the DK and the MK rumour models, and their variants, stifling is the only mechanism that results in cessation of rumour spreading. In reality, however, cessation can occur also purely as a result of spreaders forgetting to tell the rumour, or their disinclination to spread the rumour anymore. Following a suggestion in Ref. [8], we take this important mechanism into account by assuming that individuals may also cease spreading a rumour spontaneously (i.e., without the need for any contact) at a rate δ . The spreading process starts with one (or more) element(s) becoming informed of a rumour and terminates when no spreaders are left in the population.

3. IMC mean-field equations

We can describe the dynamics of the above model on a network within the framework of the IMC. The IMC was originally introduced as a means for modelling social processes involving many interacting actors (or

agents) [25]. More recently they have been applied to the dynamics of cascading failures in electrical power networks [26], and the spread of malicious software (malware) on the Internet [27]. An IMC consists of N interacting nodes, with each node having a state that evolves in time according to an internal Markov chain. Unlike conventional Markov chains, however, the corresponding internal transition probabilities depend not only on the current state of a node itself, but also on the states of all the nodes to which it is connected. The overall system evolves according to a global Markov Chain whose state space dimension is the product of states describing each node. When dealing with large networks, the exponential growth in the state space renders direct numerical solution of the IMCs extremely difficult, and one has to resort to either Monte Carlo simulations or approximate solutions. In the case of rumour model, each internal Markov chain can be in one of the three states: ignorant, spreader or stifter. For this case we derive below a set of coupled rate equations which describe on a mean-field level the dynamics of the IMC. We note that a similar mean-field approach may also be applicable to other dynamical processes on networks which can be described within the IMC framework.

Consider now a node j which is in the ignorant state at time t . We denote with p_{ii}^j the probability that this node stays in the ignorant state in the time interval $[t, t + \Delta t]$, and with $p_{is}^j = 1 - p_{ii}^j$ the probability that it makes a transition to the spreader state. It then follows that

$$p_{ii}^j = (1 - \Delta t \lambda)^g, \quad (1)$$

where $g = g(t)$ denotes the number of neighbours of node j which are in the spreader state at time t . In order to progress, we shall coarse-grain the microdynamics of the system by classifying nodes in our network according to their degree and taking statistical average of the above transition probability over degree classes.

Assuming that a node j has k links, g can be considered as a stochastic variable which has the following binomial distribution:

$$\Pi(g, t) = \binom{k}{g} \theta(k, t)^g (1 - \theta(k, t))^{k-g}, \quad (2)$$

where $\theta(k, t)$ is the probability at time t that an edge emanating from an ignorant node with k links points to a spreader node. This quantity can be written as

$$\theta(k, t) = \sum_{k'} P(k'|k) P(s_{k'}|i_k) \approx \sum_{k'} P(k'|k) \rho^s(k', t). \quad (3)$$

In this equation $P(k'|k)$ is the degree–degree correlation function, $P(s_{k'}|i_k)$ the conditional probability that a node with k' links is in the spreader state, given that it is connected to an ignorant node with degree k , and $\rho^s(k', t)$ is the density of spreader nodes at time t which belong to connectivity class k . In the above equation the final approximation is obtained by ignoring dynamic correlations between the states of neighbouring nodes.

The transition probability $\bar{p}_{ii}(k, t)$ averaged over all possible values of g is then given by

$$\begin{aligned} \bar{p}_{ii}(k, t) &= \sum_{g=0}^k \binom{k}{g} (1 - \lambda \Delta t)^g \theta(k, t)^g (1 - \theta(k, t))^{k-g} \\ &= \left(1 - \lambda \Delta t \sum_{k'} P(k'|k) \rho^s(k', t) \right)^k. \end{aligned} \quad (4)$$

In a similar fashion we can derive an expression for the probability $\bar{p}_{ss}(k, t)$ that a spreader node which has k links stays in this state in the interval $[t, t + \Delta t]$. In this case, however, we also need to compute the expected value of the number of stifter neighbours of the node at time t . Following steps similar to the previous paragraphs we obtain

$$\bar{p}_{ss}(k, t) = \left(1 - \alpha \Delta t \sum_{k'} P(k'|k) (\rho^s(k', t) + \rho^r(k', t)) \right)^k (1 - \delta \Delta t). \quad (5)$$

The corresponding probability for a transition from the spreader to the stifter state, $\bar{p}_{sr}(k, t)$ is given by $\bar{p}_{sr}(k, t) = 1 - \bar{p}_{ss}(k, t)$.

The above transition probabilities can be used to set up a system of coupled Chapman–Kolmogorov equations for the probability distributions of the population of ignorants, spreaders and stiflers within each connectivity class. However, ignoring fluctuations around expected values we can also obtain a set of deterministic rate equations for the expected values of these quantities.

Denote with $I(k, t), S(k, t), R(k, t)$ the expected values of the populations of nodes belonging to connectivity class k which at time t are in the ignorant, spreader or stifter state, respectively. The event that an ignorant node in class k will make a transition to the spreader state during $[t, t + \Delta t]$ is a Bernoulli random variable with probability $(1 - p_{ii}(k, t))$ of success. As a sum of i.i.d random Bernoulli variables, the total number of successful transitions in this time interval has a binomial distribution, with an expected value given by $I(k, t)(1 - p_{ii}(k, t))$. Hence the rate of change in the expected value of the population of ignorant nodes belonging to class k is given by

$$I(k, t + \Delta t) = I(k, t) - I(k, t) \left[1 - \left(1 - \lambda \Delta t \sum_{k'} \rho^s(k', t) P(k'|k) \right)^k \right]. \tag{6}$$

Similarly, we can write the corresponding rate of change in the population of spreaders and stiflers as follows

$$S(k, t + \Delta t) = S(k, t) + I(k, t) \left[1 - \left(1 - \lambda \Delta t \sum_{k'} \rho^s(k', t) P(k'|k) \right)^k \right] - S(k, t) \left[1 - \left(1 - \alpha \Delta t \sum_{k'} (\rho^s(k', t) + \rho^r(k', t)) P(k'|k) \right)^k (1 - \delta \Delta t) \right] \tag{7}$$

$$R(k, t + \Delta t) = R(k, t) + S(k, t) \left[1 - \left(1 - \alpha \Delta t \sum_{k'} (\rho^s(k', t) + \rho^r(k', t)) P(k'|k) \right)^k (1 - \delta \Delta t) \right] \tag{8}$$

In the above $\rho^i(k, t), \rho^s(k, t)$, and $\rho^r(k, t)$ are the fraction of nodes belonging to class k which are in the ignorant, spreader and stifter states, respectively. These quantities satisfy the normalization condition $\rho^i(k, t) + \rho^s(k, t) + \rho^r(k, t) = 1$. In the limit $\Delta t \rightarrow 0$ we obtain

$$\frac{\partial \rho^i(k, t)}{\partial t} = -k \lambda \rho^i(k, t) \sum_{k'} \rho^s(k', t) P(k'|k), \tag{9}$$

$$\frac{\partial \rho^s(k, t)}{\partial t} = k \lambda \rho^i(k, t) \sum_{k'} \rho^s(k', t) P(k'|k) - k \alpha \rho^s(k, t) \sum_{k'} (\rho^s(k', t) + \rho^r(k', t)) P(k'|k) - \delta \rho^s(k, t), \tag{10}$$

$$\frac{\partial \rho^r(k, t)}{\partial t} = k \alpha \rho^s(k, t) \sum_{k'} (\rho^s(k', t) + \rho^r(k', t)) P(k'|k) + \delta \rho^s(k, t). \tag{11}$$

For future reference we note here that information on the underlying network is incorporated in the above equations solely via the degree–degree correlation function. Thus, in our analytical and numerical studies reported in the next section we do not need to generate any actual network. All that is required is either an analytical expression for $P(k'|k)$ or a numerical representation of this quantity.

4. Analysis

4.1. Homogeneous networks

In order to understand some basic features of our rumour model we first consider the case of homogeneous networks, in which degree fluctuations are very small and there are no degree correlations. In this case the rumour equations become:

$$\frac{d\rho^i(t)}{dt} = -\lambda\bar{k}\rho^i(t)\rho^s(t), \quad (12)$$

$$\frac{d\rho^s(t)}{dt} = \lambda\bar{k}\rho^i(t)\rho^s(t) - \alpha\bar{k}\rho^s(t)(\rho^s(t) + \rho^r(t)) - \delta\rho^s(t), \quad (13)$$

$$\frac{d\rho^r(t)}{dt} = \alpha\bar{k}\rho^s(t)(\rho^s(t) + \rho^r(t)) + \delta\rho^s(t), \quad (14)$$

where \bar{k} denotes the constant degree distribution of the network (or the average value for networks in which the probability of finding a node with a different connectivity decays exponentially fast).

The above system of equations can be integrated analytically using a standard approach. In the infinite time limit, when $\rho^s(\infty) = 0$, we obtain the following transcendental equation for $R = \rho^r(\infty)$, the final fraction of nodes which ever hear the rumour (we call this the final rumour size)

$$R = 1 - e^{-\varepsilon R}, \quad (15)$$

where

$$\varepsilon = \frac{(\alpha + \lambda)\bar{k}}{\delta + \alpha\bar{k}}. \quad (16)$$

Eq. (15) admits a non-zero solution only if $\varepsilon > 1$. For $\delta \neq 0$ this condition is fulfilled provided

$$\frac{\lambda}{\delta}\bar{k} > 1, \quad (17)$$

which is precisely the same threshold condition as found in the SIR model of epidemic spreading on homogeneous networks [28,29]. On the other hand, in the special case $\delta = 0$ (i.e., when the forgetting mechanism is absent) $\varepsilon = 1 + \lambda/\alpha > 1$, and so Eq. (15) always admits a non-zero solution, in agreement with the result in Ref. [22].

The above result shows, however, that the presence of a forgetting mechanism results in the appearance of a finite threshold in the rumour spreading rate below which rumours cannot spread in homogeneous networks. Furthermore, the value of the threshold is *independent* of α (i.e., the stifling mechanism), and is the same as that for the SIR model of epidemics on such networks. This result can be understood by noting that in the above equations the terms corresponding to the stifling mechanism are of second order in ρ^s , while the terms corresponding to the forgetting mechanism are only of first order in this quantity. Thus in the initial state of the spreading process, where $\rho^s \approx 0$ and $\rho^r \approx 0$, the effect of stifling is negligible relative to that of forgetting, and the dynamics of the model reduces to that of the SIR model.

4.2. Inhomogeneous networks

Next we consider uncorrelated inhomogeneous networks. In such networks the degree–degree correlations can be written as (Ref. [30]):

$$P(k'|k) = q(k') = \frac{k'P(k')}{\langle k \rangle}, \quad (18)$$

where $P(k')$ is the degree distribution and $\langle k \rangle$ is the average degree. In this case the dynamic of rumour spreading is describe by Eqs. (9)–(11). Eq. (9) can be integrated exactly to yield

$$\rho^i(k, t) = \rho^i(k, 0)e^{-\lambda k \phi(t)}, \quad (19)$$

where $\rho^i(k, 0)$ is the initial density of ignorant nodes with connectivity k , and we have introduced the auxiliary function

$$\phi(t) = \sum_k q(k) \int_0^t \rho^s(k, t') dt' \equiv \int_0^t \langle \langle \rho^s(k, t') \rangle \rangle dt'. \tag{20}$$

In the above equation and hereafter we use the shorthand notation

$$\langle \langle O(k) \rangle \rangle = \sum_k q(k) O(k) \tag{21}$$

with

$$q(k) = \frac{kP(k)}{\langle k \rangle}. \tag{22}$$

In order to obtain an expression for the final size of the rumour, R , it is more convenient to work with ϕ . Assuming an homogeneous initial distribution of ignorants, $\rho^i(k, 0) = \rho_0^i$, we can obtain a differential equation for this quantity by multiplying Eq. (10) with $q(k)$ and summing over k . This yields after some elementary manipulations:

$$\frac{d\phi}{dt} = 1 - \langle \langle e^{-\lambda k \phi} \rangle \rangle - \delta \phi - \alpha \int_0^t [1 - \langle \langle e^{-\lambda k \phi(t')} \rangle \rangle] \langle \langle k \rho^s(k, t') \rangle \rangle dt', \tag{23}$$

where, without loss of generality, we have also put $\rho_0^i \approx 1$.

In the limit $t \rightarrow \infty$ we have $d\phi/dt = 0$, and Eq. (23) becomes:

$$0 = 1 - \langle \langle e^{-\lambda k \phi_\infty} \rangle \rangle - \delta \phi_\infty - \alpha \int_0^\infty [1 - \langle \langle e^{-\lambda k \phi(t')} \rangle \rangle] \langle \langle k \rho^s(k, t') \rangle \rangle dt', \tag{24}$$

where $\phi_\infty = \lim_{t \rightarrow \infty} \phi(t)$.

For $\alpha = 0$ Eq. (24) can be solved explicitly to obtain Φ_∞ [28]. For $\alpha \neq 0$ we solve (24) to leading order in α . Integrating Eq. (10) to zero order in α we obtain

$$\rho^s(k, t) = 1 - e^{-\lambda k \phi} - \delta \int_0^t e^{\delta(t-t')} [1 - e^{-\lambda k \phi(t')}] dt' + O(\alpha). \tag{25}$$

Close to the critical threshold both $\phi(t)$ and ϕ_∞ are small. Writing $\phi(t) = \phi_\infty f(t)$, where $f(t)$ is a finite function, and working to leading order in ϕ_∞ , we obtain

$$\rho^s(k, t) \simeq -\delta \lambda k \phi_\infty \int_0^t e^{\delta(t-t')} f(t') dt' + O(\phi_\infty^2) + O(\alpha). \tag{26}$$

Inserting this in Eq. (24) and expanding the exponential to the relevant order in ϕ_∞ we find

$$0 = \phi_\infty [\lambda \langle \langle k \rangle \rangle - \delta - \lambda^2 \langle \langle k^2 \rangle \rangle (1/2 + \alpha \langle \langle k \rangle \rangle I) \phi_\infty] + O(\alpha^2) + O(\phi_\infty^3), \tag{27}$$

where I is a finite and positive-defined integral. The non-trivial solution of this equation is given by

$$\phi_\infty = \frac{\lambda \langle \langle k \rangle \rangle - \delta}{\lambda^2 \langle \langle k^2 \rangle \rangle (\frac{1}{2} + \alpha I \langle \langle k \rangle \rangle)}. \tag{28}$$

Noting that $\langle \langle k \rangle \rangle = \langle k^2 \rangle / \langle k \rangle$ and $\langle \langle k^2 \rangle \rangle = \langle k^3 \rangle / \langle k \rangle$ we obtain

$$\phi_\infty = \frac{2 \langle k \rangle \left(\frac{\langle k^2 \rangle}{\langle k \rangle} \lambda - \delta \right)}{\lambda^2 \langle k^3 \rangle \left(1 + 2\alpha I \frac{\langle k^2 \rangle}{\langle k \rangle} \right)}. \tag{29}$$

This yields a positive value for ϕ_∞ provided that

$$\frac{\lambda}{\delta} \geq \frac{\langle k \rangle}{\langle k^2 \rangle}. \quad (30)$$

Thus, to leading order in α , the critical rumour threshold is independent of the stifling mechanism and is the same as for the SIR model [28,29]. In particular, for $\delta = 1$ the critical rumour spreading threshold is given by $\lambda_c = \langle k \rangle / \langle k^2 \rangle$, and Eq. (29) simplifies to

$$\phi_\infty = \frac{2\langle k \rangle (\lambda - \lambda_c)}{\lambda^2 \langle k^3 \rangle (\lambda_c + 2\alpha I)}. \quad (31)$$

Finally, R is given by

$$R = \sum_k P(k) (1 - e^{-\lambda k \phi_\infty}). \quad (32)$$

The solution to the above equation depends on the form of $P(k)$. In particular, for homogeneous networks where all the moments of the degree distribution are bounded, we can expand the exponential in Eq. (32) to obtain

$$R \approx \sum_k P(k) \lambda k \phi_\infty = \frac{2\langle k \rangle^2 (\lambda - \lambda_c)}{\lambda \langle k^3 \rangle (\lambda_c + 2\alpha I)}, \quad (33)$$

which shows that $R \sim (\lambda - \lambda_c)$ in the vicinity of the rumour threshold. For heterogeneous networks, one must solve (32) explicitly. This can be done for example, as for the SIR model [28].

5. Numerical results

5.1. Random graphs and uncorrelated scale-free networks

We consider first *uncorrelated* networks, for which the degree–degree correlations are given by Eq. (18). We shall consider two classes of such networks. The first class is the Erdős–Rényi random networks, which for large N have a Poisson degree distribution:

$$P(k) = e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}. \quad (34)$$

The above degree distribution peaks at an average value $\langle k \rangle$ and shows small fluctuations around this value. The second class we consider are scale-free networks which have a power law degree distribution:

$$P(k) = \begin{cases} Ak^{-\gamma} & k_{min} \leq k \\ 0 & \text{otherwise.} \end{cases} \quad (35)$$

In the above equation k_{min} is the minimum degree of the networks and A is a normalization constant. Recent studies of social networks on the Internet indicates that many of these networks show highly right-skewed degree distributions, which could often be modelled by a power-law degree distribution [15,17,18]. For $2 \leq \gamma \leq 3$ the variance of the above degree distribution becomes infinite in the limit of infinite system size while the average degree distribution remains finite. We shall consider henceforth SF networks with $\gamma = 3$.

Our studies of uncorrelated networks were performed using the above forms of $P(k)$ to represent ER and SF networks, respectively. The size of the networks considered was $N = 10^5$ and 10^6 , and the average degree was fixed at $\langle k \rangle = 7$. For each network considered we generated a sequence of N random integers distributed according to its degree distribution. The coupled set of differential (9)–(11) were then solved numerically using a standard finite difference scheme, and numerical convergence with respect to the step size was checked numerically. In the following and throughout the paper all calculations reported are performed by starting the rumour from a randomly chosen initial spreader, and averaging the results over 300 runs with different initial spreaders. The calculations reported below were performed for networks consisting of $N = 10^6$ nodes.

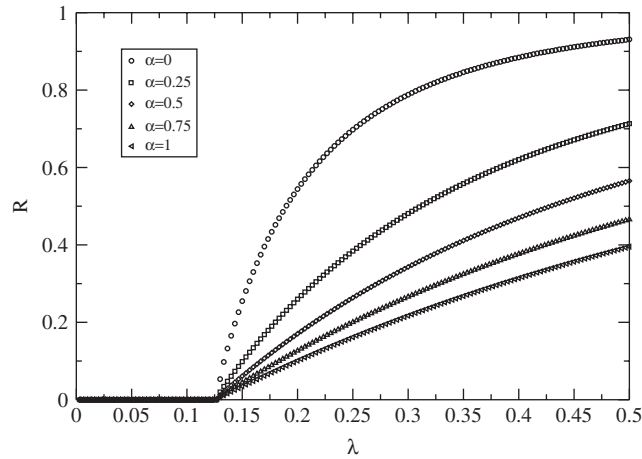


Fig. 1. The final size of the rumour, R is shown as a function of the spreading rate λ for the ER network of size 10^6 . The results are shown for several values of the stifling parameter α .

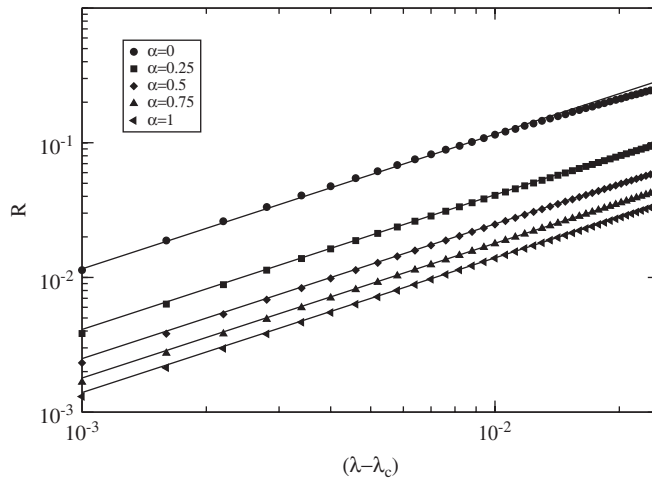


Fig. 2. R is plotted as a function of $\lambda - \lambda_c$ for the ER network of size 10^6 , using different values of α . Solid lines show our numerical fits to the form $R \sim (\lambda - \lambda_c)^\beta$, with $\beta = 1$.

In our first set of calculations we set $\delta = 1$ and investigate the dynamics as a function of the rumour spreading rate λ and the stifling rate α . First we focus on the impact of network topology on the final size of a rumour, R , which for inhomogeneous networks is obtained from

$$R = \sum_k \rho^r(k, t_\infty), \tag{36}$$

where t_∞ denotes a sufficiently long time at which the spreading process has reached its steady state (i.e., no spreaders are left in the network). In Fig. 1 R corresponding to the ER network is plotted as a function of λ , and for several different values of the stifling parameter α . It can be seen that in this network R exhibits a critical threshold λ_c below which a rumour cannot spread in the network. Furthermore, just as in the case of homogeneous networks, the value of the threshold does not depend on α , and is at $\lambda_c = 0.12507$. This value is in excellent agreement with the analytical results obtained in the previous section. We also verified numerically that, in agreement with our analytical findings, the behaviour of R in the vicinity of the critical point can be described with the form

$$R \sim A(\lambda - \lambda_c), \tag{37}$$

where $A = A(\alpha)$ is a smooth and monotonically decreasing function of α . The results are shown in Fig. 2 where R is plotted as function of λ for a range of values of α , together with the corresponding fits.

Next we turn to our results for the SF network. In Fig. 3 results for R in this network are shown. In this case we also observe the presence of an α -independent rumour threshold, albeit for much smaller spreading rates than for the ER network. We have verified numerically that in this case the threshold is approached with zero slope, as can also be gleaned from Fig. 3. Since the value of the threshold is independent of α , we can use the well-known result for the SIR model (the $\alpha = 0$ case) to conclude that in the limit of infinite system size the threshold seen in the SF network will approach zero. It is therefore not an intrinsic property of rumour spreading on this network.

In order to further analyse the behaviour of R in SF networks, we have numerically fitted our results to the stretched exponential form,

$$R \sim \exp(-C/\lambda), \tag{38}$$

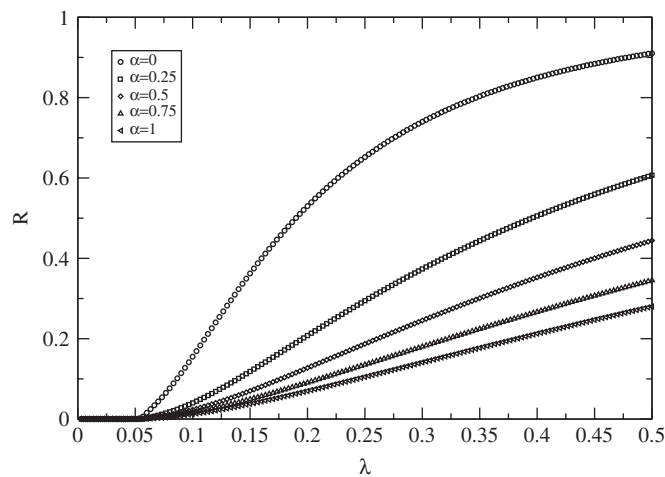


Fig. 3. The final size of the rumour, R is shown as a function of the spreading rate λ for the SF network of size 10^6 . The results are shown for several values of the stifling parameter α .

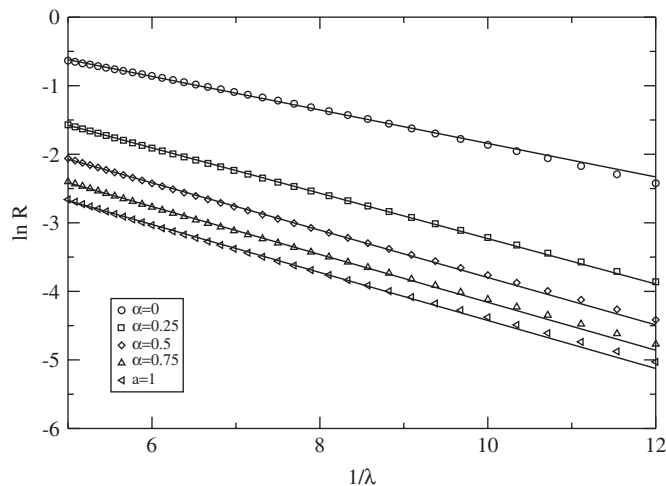


Fig. 4. R (in log scale) in the SF network of size 10^6 is plotted as a function of $1/\lambda$ and several values of α . Solid lines are our numerical fits to the stretched exponential form $R = B(\alpha) \exp(-C(\alpha)/\lambda)$.

with C depending only weakly on α . This form was found to describe the epidemic prevalence in both the SIS and the SIR model of epidemics [28,31]. The results are displayed in Fig. 4, and they clearly indicate that the stretched exponential form also nicely describes the behaviour of R in our rumour model. This result provides further support for our conjecture that the general rumour model does not exhibit any threshold behaviour on SF networks (at least in the limit of infinite systems size).

In addition to investigating the impact of network topology on the steady-state properties of the model, it is of great interest to understand how the time-dependent behaviour of the model is affected by topology. In Figs. 5 and 6 we display, as representative examples, time evolution of, respectively, the total fractions of stiflers and spreaders, in both networks for $\lambda = 1$ and two sets of values of the cessation parameters: $\{\delta = 1, \alpha = 0\}$, and $\{\delta = 0, \alpha = 1\}$. The first set of parameters corresponds to a spreading process in which cessation results purely from spontaneous forgetting of a rumour by spreaders, or their disinclination to spread the rumour any further. The second set corresponds to a scenario where individuals keep spreading the rumour until they become stiflers due to their contacts with other spreaders or stiflers in the network. As can be seen in Fig. 5, in the first scenario the initial spreading rate of a rumour on the SF network is much faster

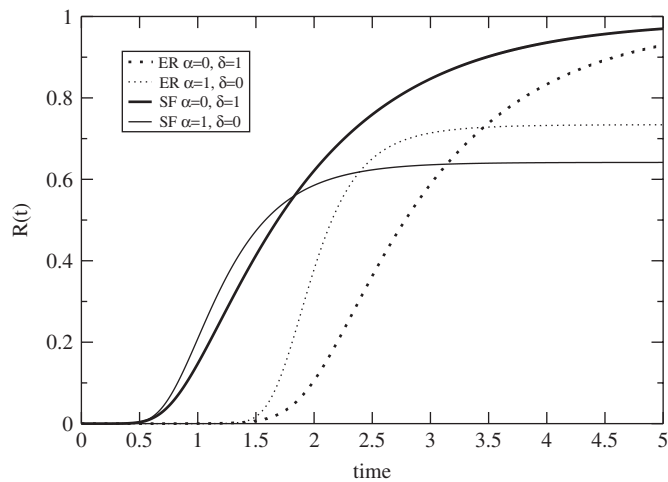


Fig. 5. Time evolution of the density of stiflers is shown on the ER (dashed lines) and the SF network (solid lines) when the dynamics starts with a single spreader node. Results are shown for two sets of values of the cessation parameters $\{\alpha = 0, \delta = 1\}$ and $\{\alpha = 1, \delta = 0\}$. The network sizes are $N = 10^6$.

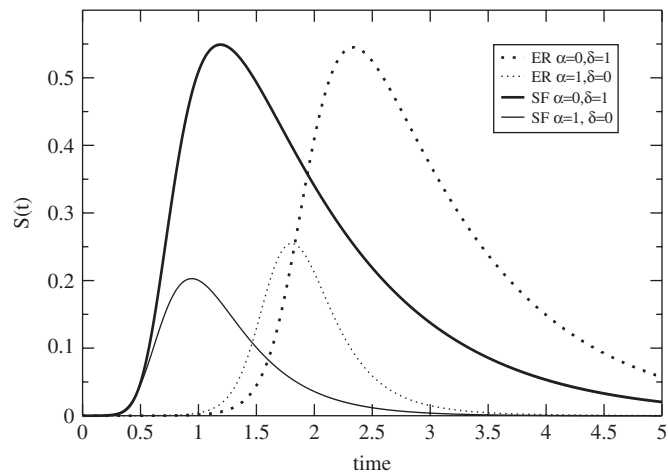


Fig. 6. Time evolution of the density of spreaders is shown for the same networks, model parameters and initial conditions as in Fig. 5.

than on the ER network. In fact, we find that the time required for the rumour to reach 50% of nodes in the ER random graph is nearly twice as long as the corresponding time on the SF networks. This large difference in the spreading rate is due to the presence of highly connected nodes (social hubs) in the SF network, whose presence greatly speeds up the spreading of a rumour. We note that in this scenario not only a rumour spreads initially faster on SF networks, but it also reaches a higher fraction of nodes at the end of the spreading process.

It can be seen from Figs. 5 and 6 that in the second spreading scenario (i.e., when stifling is the only mechanism for cessation) the initial spreading rate on the SF network is, once again, higher than on the ER network. However, unlike the previous situation, the ultimate size of the rumour is higher on the ER network. This behaviour is due to the conflicting roles that hubs play when the stifling mechanism is switched on. Initially the presence of hubs speeds up the spreading but once they turn into stiflers they also effectively impede further spreading of the rumour.

5.2. Assortatively correlated scale-free networks

Recent studies have revealed that social networks display assortative degree correlations, implying that highly connected vertices preferably connect to vertices which are also highly connected [23]. In order to study the impact of such correlations on the dynamics of our model, we make use of the following ansatz for the degree–degree correlation function [32]

$$P(k'|k) = (1 - \beta)q(k') + \beta\delta_{kk'}; \quad (0 \leq \beta < 1). \quad (39)$$

The above form allows us to study in a controlled way the impact of degree correlations on the spreading of rumour.

Using the above degree–degree correlation function we numerically solved Eqs. (9)–(11) for a SF network characterized by $\gamma = 3$ and $\langle k \rangle = 7$. The network size was fixed at $N = 100,000$, and we used two values for the correlation parameter: $\beta = 0.2$ and $\beta = 0.4$. Fig. 7 displays R as a function of λ , and for $\alpha = 0.5, 0.75, 1$ (the value of δ was fixed at 1).

It can be seen that below $\lambda \approx 0.5$ a rumour will reach a somewhat smaller fraction of nodes on the correlated networks than on the uncorrelated ones. However, for larger values of λ this behaviour reverses, and the final size of the rumour in assortatively correlated networks shows a higher value than in the uncorrelated network. We thus conclude that the qualitative impact of degree correlations on the final size of a rumour depends very much on the rumour spreading rate.

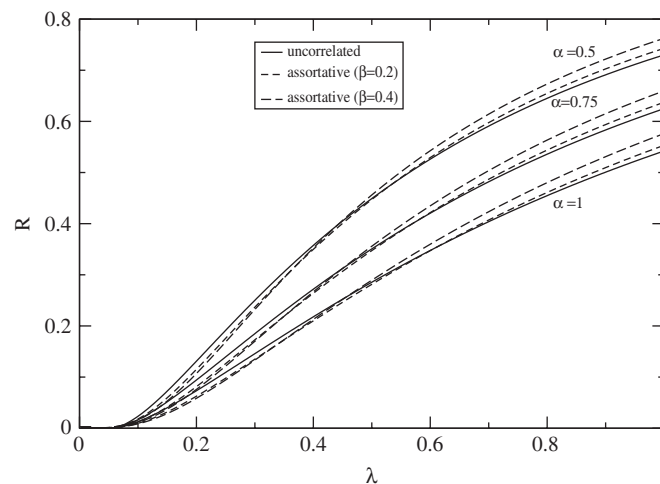


Fig. 7. The final size of the rumour is plotted as a function of λ and for several values of α in the SF network of size 10^5 . Results are shown in the absence (solid lines) of assortative degree–degree correlations and in the presence of such correlations. The correlation strengths used are $\beta = 0.2$ (short dashed lines) and $\beta = 0.4$ (long dashed lines).

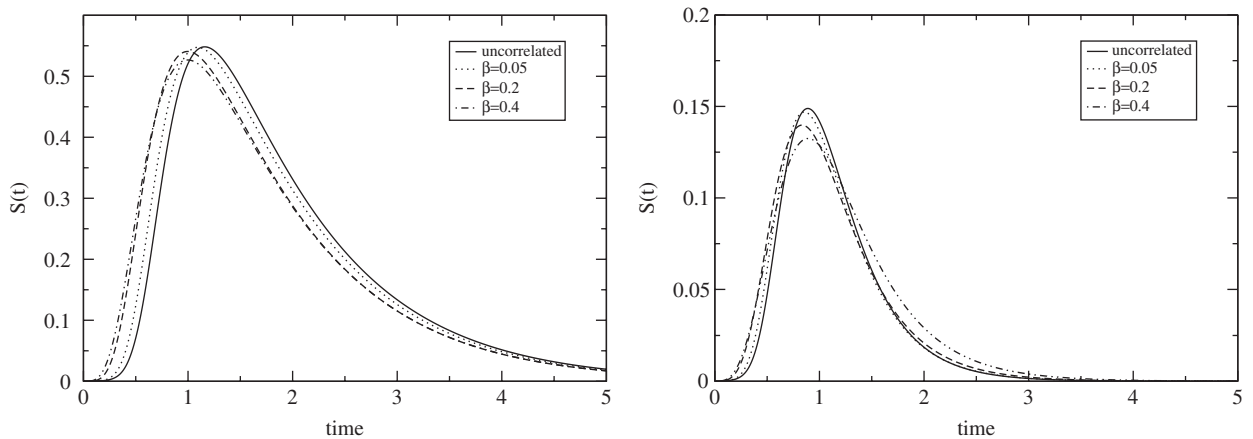


Fig. 8. The impact of assortative correlations on time evolution of the density of rumour spreaders. Results are shown for the SF networks of size $N = 10^5$ and several values of the correlation strength, β . The left panel shows results using $\alpha = 0$ and the right panel those using $\alpha = 1$.

Finally, we investigate the effect of assortative correlations on the speed of rumour spreading. In Fig. 8 we show our results for the time evolution of the total fraction of spreaders, $S(t)$, in scale-free networks consisting of $N = 100,000$ nodes and for a correlation strength ranging from $\beta = 0$ to 0.4. In these calculations the value of λ was fixed at 1, and we considered two values of α : 0,1. It can be seen that the initial rate at which a rumour spreads increases with an increase in the strength of assortative correlations regardless of the value of α . However, for $\alpha = 1$ the rumour also dies out faster when such correlations are stronger.

6. Conclusions

In this paper we introduced a general model of rumour spreading on complex networks. Unlike previous rumour models, our model incorporates two distinct mechanisms that cause cessation of a rumour, stifling and forgetting. We used an IMC formulation of the model to derive deterministic mean-field equations for the dynamics of the model on complex networks. Using these equations, we investigated analytically and numerically the behaviour of the model on ER random graphs and scale-free networks with exponent $\gamma = 3$. The critical behaviour, the dynamics and the stationary state of our model on these networks are significantly different from the results obtained for the dynamics of simpler rumour models on complex networks [19–22]. In particular, our results show the presence of a critical threshold in the rumour spreading rate below which a rumour cannot spread in ER networks. The value of this threshold was found to be independent of the stifling mechanism, and to be the same as the critical infection rate of the SIR epidemic model. Such a threshold is also present in the finite-size SF networks we studied, albeit at a much smaller value. However, in SF networks this threshold is reached with a zero slope and its value becomes vanishingly small in the limit of infinite network size. We also found the initial rate of spreading of a rumour to be much higher on scale-free networks than on ER random graphs. An effect which is caused by the presence of hubs in these networks, which efficiently disseminate a rumour once they become informed. Our results show that SF networks are prone to the spreading of rumours, just as they are to the spreading of infections.

Finally, we used a local ansatz for the degree–degree correlation function in order to numerically investigate the impact of assortative degree correlations on the speed of rumour spreading on SF networks. These correlations were found to speed up the initial rate of spreading in SF networks. However, their impact on the final fraction of nodes which hear a rumour depends very much on the rate of rumour spreading.

In the present work we assumed the underlying network to be static, i.e., a time-independent network topology. In reality, however, many social and communication networks are highly dynamic. An example of such time-dependent social networks is Internet chatrooms, where individuals continuously make new social contacts and break old ones. Modelling spreading processes on such dynamic networks is highly challenging,

in particular, when the time scale at which network topology changes becomes comparable with the time scale of the process dynamics. We aim to tackle this highly interesting problem in future work.

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