Spatial Model of Segregation

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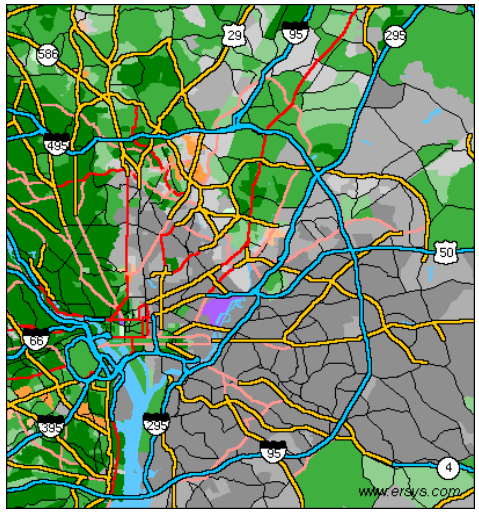
Spatial model of segregation

"Dynamic Models of Segregation", Thomas Schelling, 1971

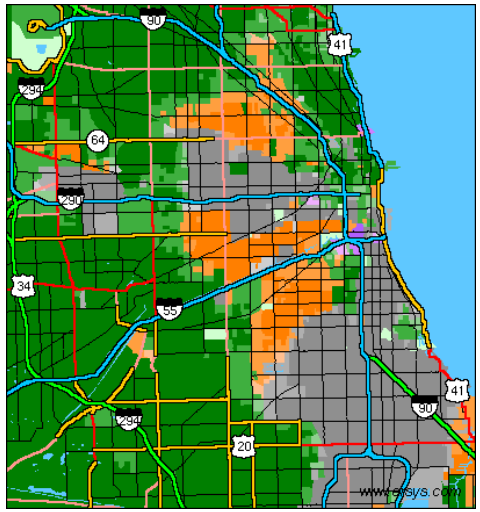
- Micromotives and macrobehavior
- Personal preferences lead to collective actions
- Global patterns of spatial segregation from homophily at a local level
- Segregated race, ethnicity, native language, income
- Cities are strongly racially segregated. Are people that racists?
- Agent based modeling: agents, rules (dynamics), aggregation

Racial segregation

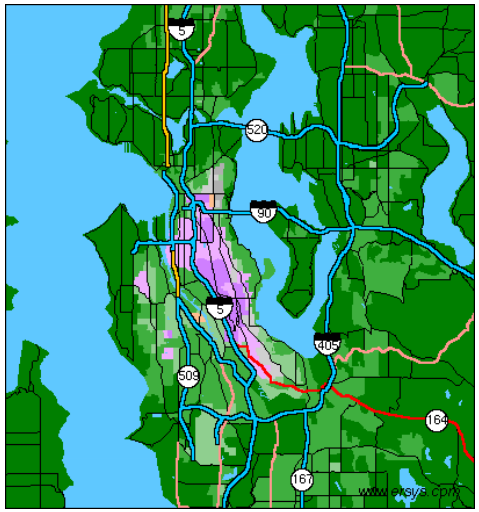
New York



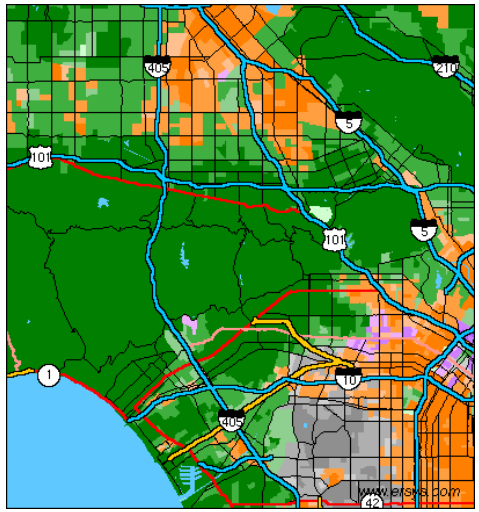
Washington



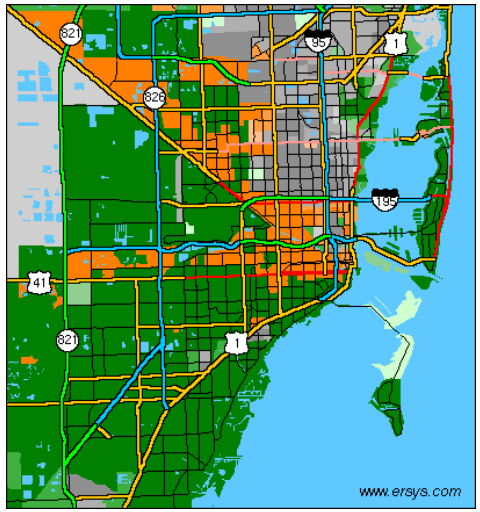
Chicago



Seattle

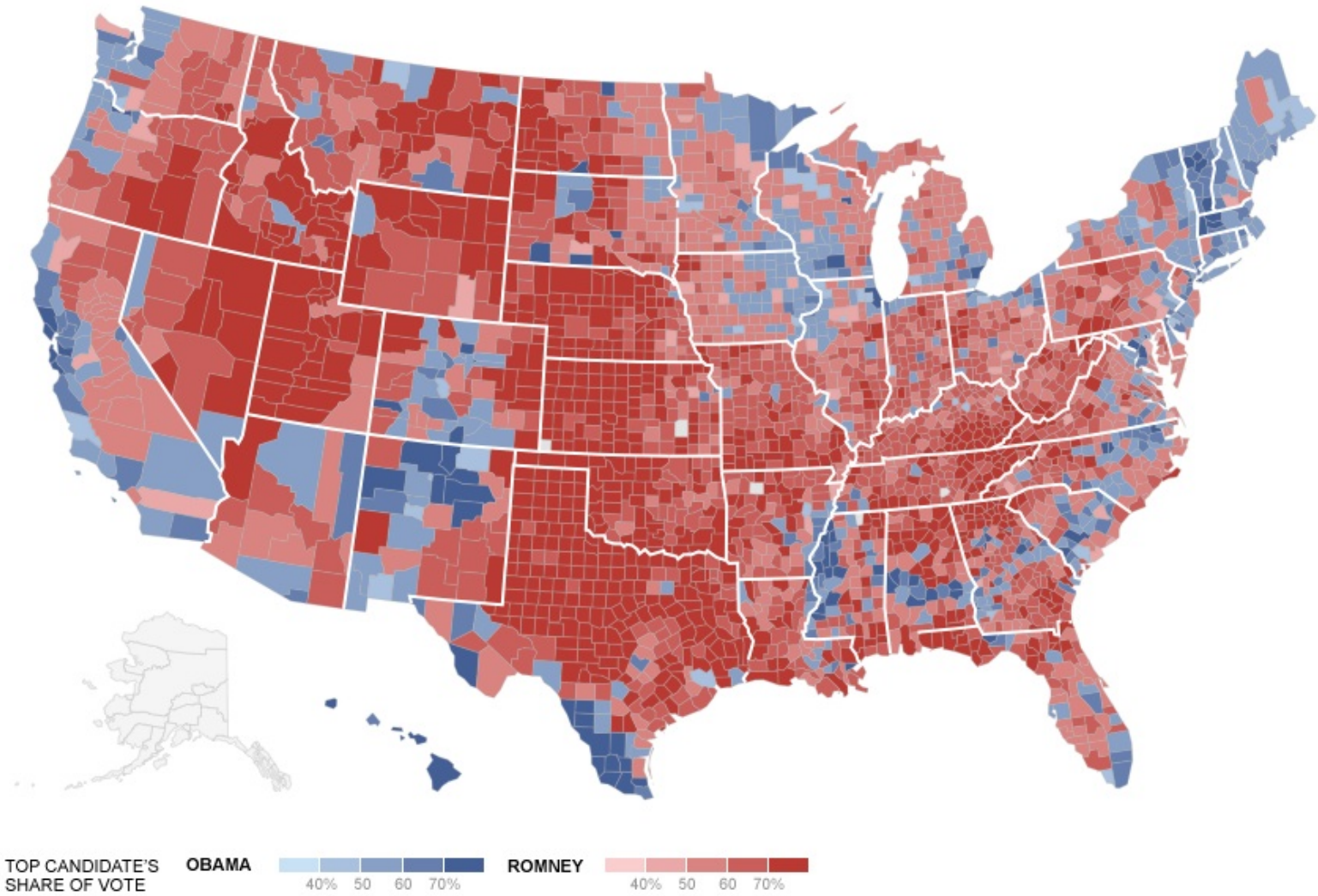


Los Angeles



Miami

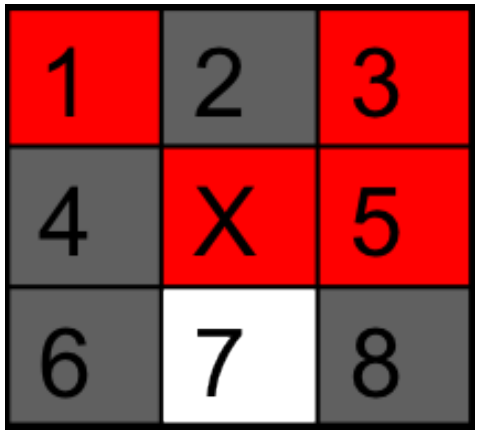
2012 US Presedential Elections Map



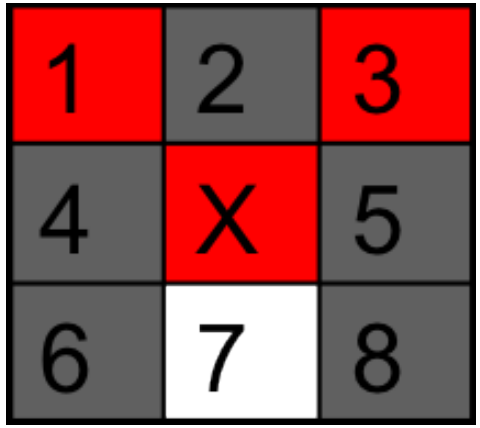
Schelling's model of segregation

- Population consitst of 2 types of agents
- Agent reside in the cells of the grid (2-dimensional geography of a city), 8 neighbors
- Some cells contain agents, some unpopulated
- Every agent wants to have at least some fraction of agents (threshold) of his type as neighbours (satisfied agent)
- On every round every unsatisfied agent moves to a satisfactory empty cell.
- Continues until everone is satisfied or can't move

Spatial segregation



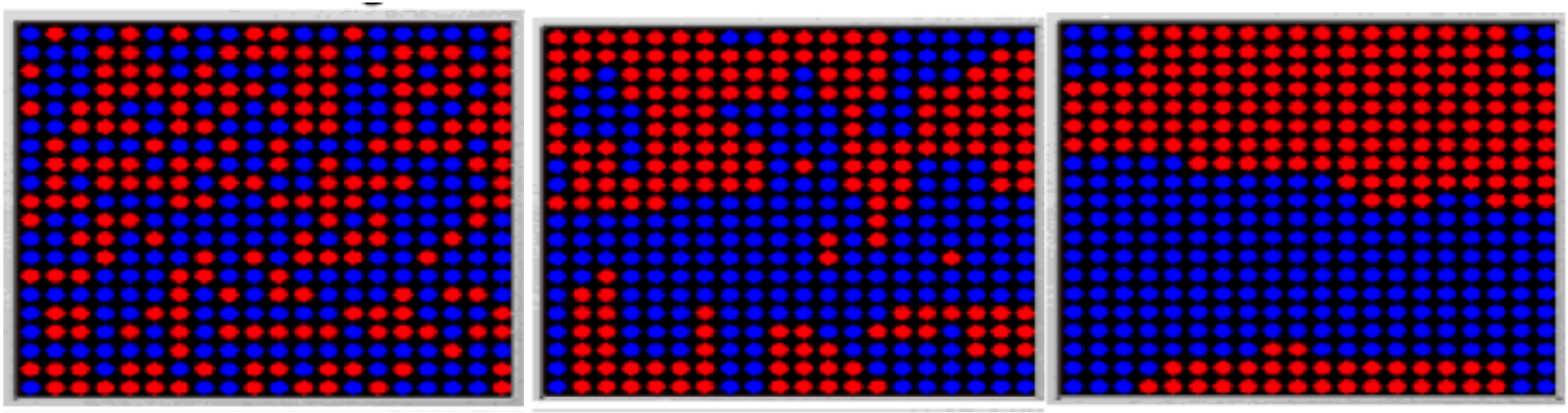
satisfied agent



unsatisfied agent

- preference thresholod $\lambda = 3/7$

Spatial segregation



Model

- N - nodes, θ - fraction of occupied by A and B

$$n_A + n_B = \theta \cdot N$$

- Share of "foreign" nearest neighbors, $k_i = \#NN$

$$P_i = \begin{cases} \#N_B/k_i, \text{if} \quad i \in A \\ \#N_A/k_i, \text{if} \quad i \in B \end{cases}$$

- Utility function, λ - sensitivity (threshold) level

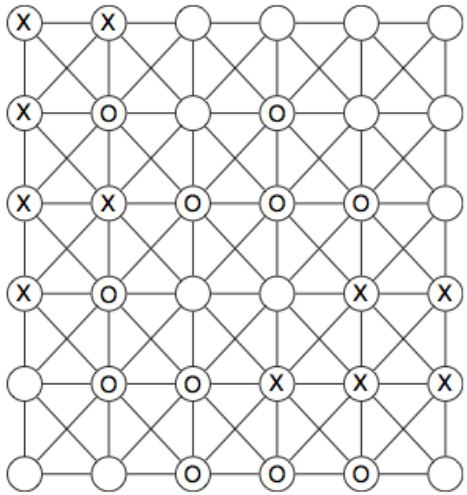
$$u_i = \begin{cases} 1, \text{if} \quad P_i \leq \lambda \\ 0, \text{if} \quad P_i > \lambda \end{cases}$$

- Every node moves to maximize its utility

Spatial segregation

x	x				
x	o		o		
x	x	o	o	o	
x	o			x	x
	o	o	x	x	x
		o	o	o	

(a)



(b)

Algorithm

- time steps $1..T$
- At every time step randomly select an agent, compute utility
- If itility is $u = 0$ move to an empty location to maximize utility
- Movements: 1) random location 2) nearest available location
- Repeat until either all utilities are maximized $\sum_i u_i = \theta N$ or reaches "frozen"state, no place to move, then $\sum_i u_i < \theta N$
- Total utility of society

$$U = \sum_i u_i$$

Measuring segregation

- Schilling's solid mixing index

$$M = \frac{1}{n_A + n_B} \sum_i P_i$$

- Freeman's segregation index

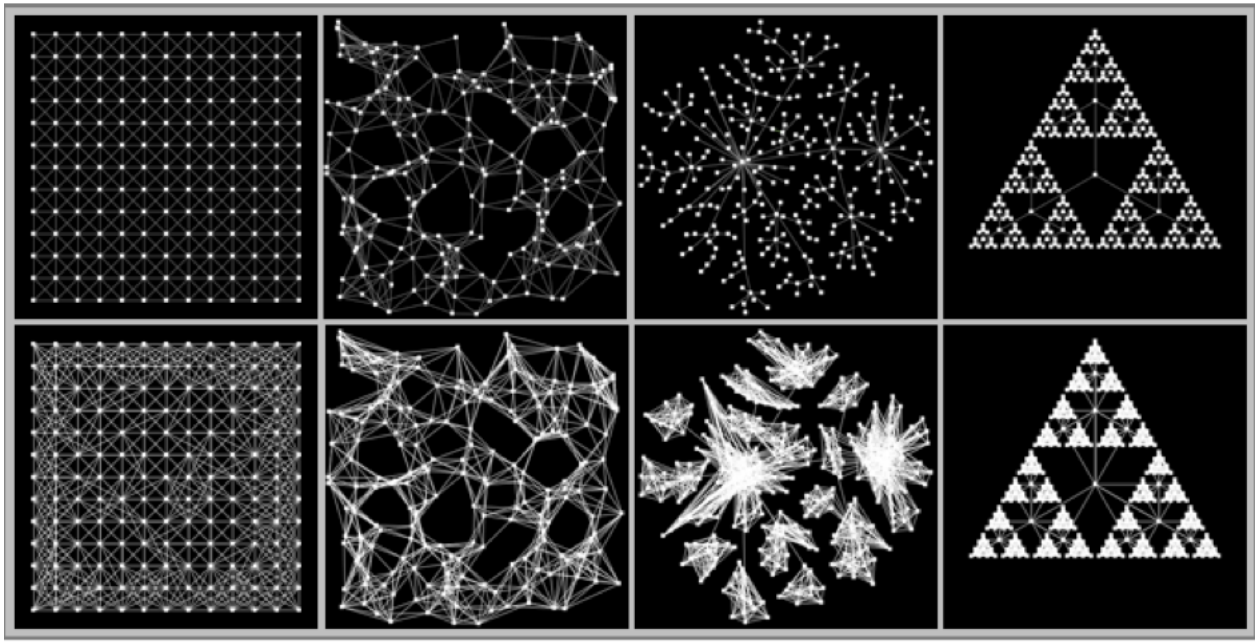
$$F = 1 - \frac{e^*}{E(e^*)}$$

$e^* = \frac{e_{AB}}{(e_{AB}+e_{AA}+e_{BB})}$ - observed proportion of between group ties,
$E(e^*) = \frac{2n_A n_B}{(n_A+n_B)(n_A+n_B-1)}$ - expected proportion for random ties

- Assortative mixing

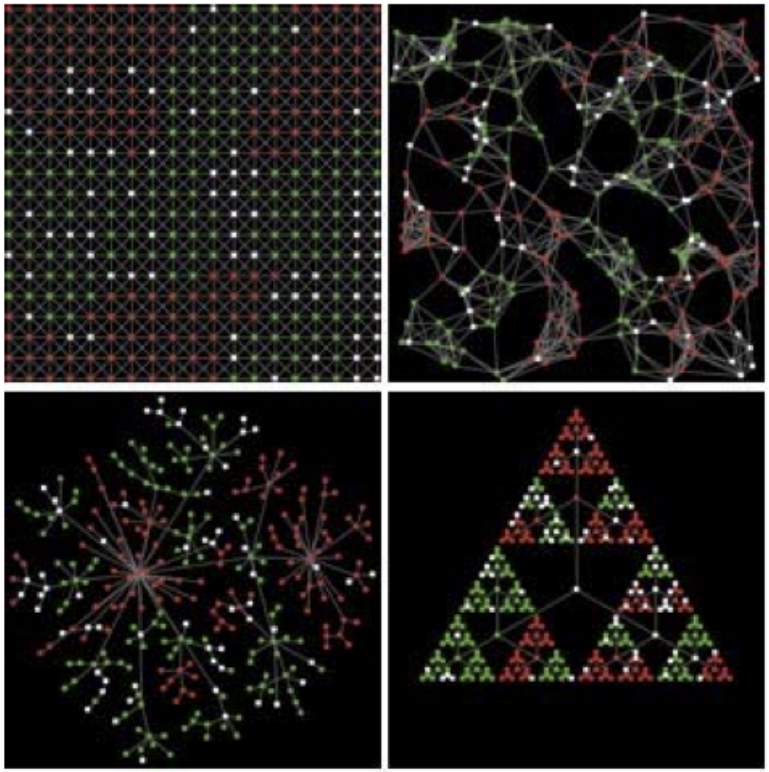
$$Q = \frac{1}{2m} \sum_{ij} (A_{ij} - \frac{k_i k_j}{2m}) \delta(c_i, c_j)$$

Spatial segregation on networks



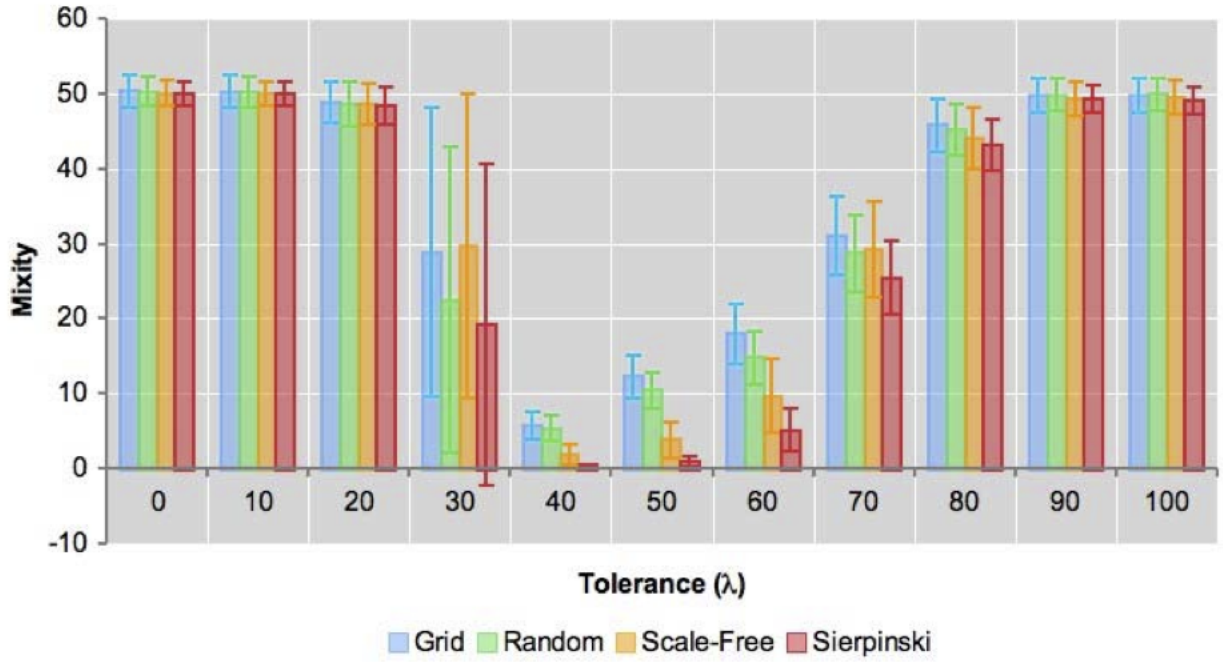
Banos, 2010

Spatial segregation on networks



Banos, 2010

Spatial segregation on networks



Banos, 2010

Model interpretation

- Spatial segregation is taking place even though no individual agent is actively seeking it
- Network structure does affect segregation
- Fixed characteristics (race) can become correlated with mutable (location)

References

- Dynamic Models of Segregation, Thomas C. Schelling, 1971
- Segregation in Social Networks, Linton Freeman, 1978
- Network effects in Schellin's model of segregation: new evidences from agent-based simulations, Arnaud Banos, 2010