Strategic Network Formation

Leonid E. Zhukov

School of Applied Mathematics and Information Science
National Research University Higher School of Economics

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Strategic network formation

M. Jackson, A. Wolinksy, 1996
"A Strategic Model of Social and Economic Networks"

- why networks becomes the way they are
- people (agents) making rational choices establishing connections
- maximizing individual utility (incentives)
- connections brings costs and benefits
- stability of the network
- social efficency (best for the society)
- friendship, professional, political, trade networks

Utility functions

- Distance-based utility (positive externalities)

$$u_i(G) = \sum_{j \in N_i^{n-1}} \delta^{l_{ij}} - d_i \cdot c$$

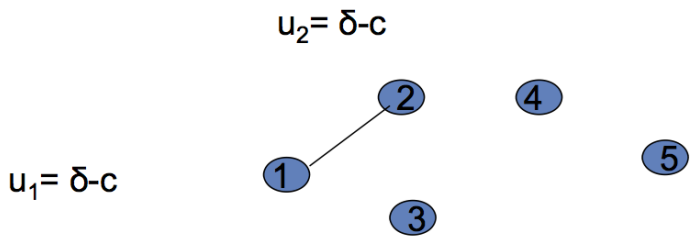
l_{ij} - shortest path, d_i - node degree, δ, C - parameters, $\delta < 1$

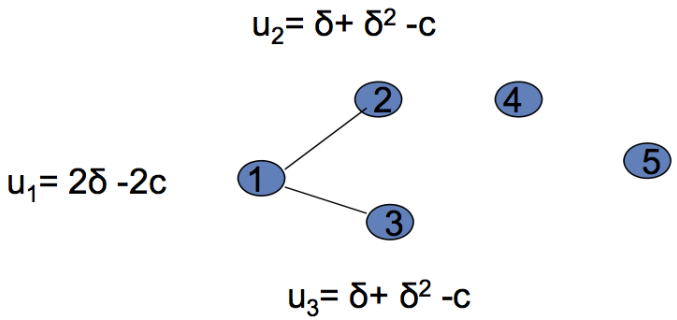
- Co-author model (negative externalities)

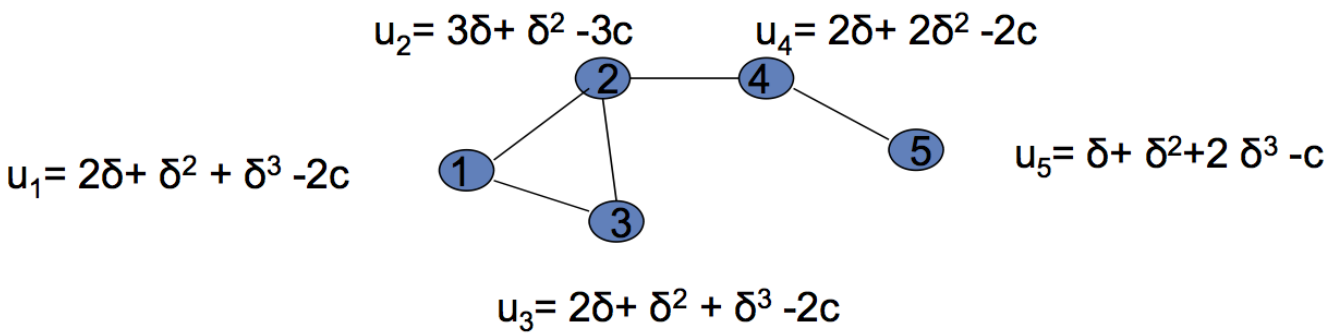
$$u_i(G) = \sum_{j \in N_i} \left(\frac{1}{d_i} + \frac{1}{d_j} + \frac{1}{d_i d_j}\right) = 1 + \sum_{j \in N_i} \left(\frac{1}{d_j} + \frac{1}{d_i d_j}\right), \quad d_i \neq 0$$

$u_i(G) = 1$ if $d_i = 0$, d_i - node degree

Distance-based utility function







Network Stability

- Evolution: forming a link - mutual consent, removing a link - one person decision
- Network is pairwise stable if no palyer wants to remove a link and no two players want to add a link:

$$\begin{aligned} \forall i \; u_i(G) &\geq u_i(G - e_{ij}) \\ \forall i,j \text{ if } u_i(G + e_{ij}) &> u_i(G), \text{then } u_j(G + e_{ij}) < u_j(G) \end{aligned}$$

- Weak condition:
  1) considers removing one link at a time
  2) considers only forming one pair at a time

Network Efficiency

- Strong Efficiency, "best network", maximize total utility for the society

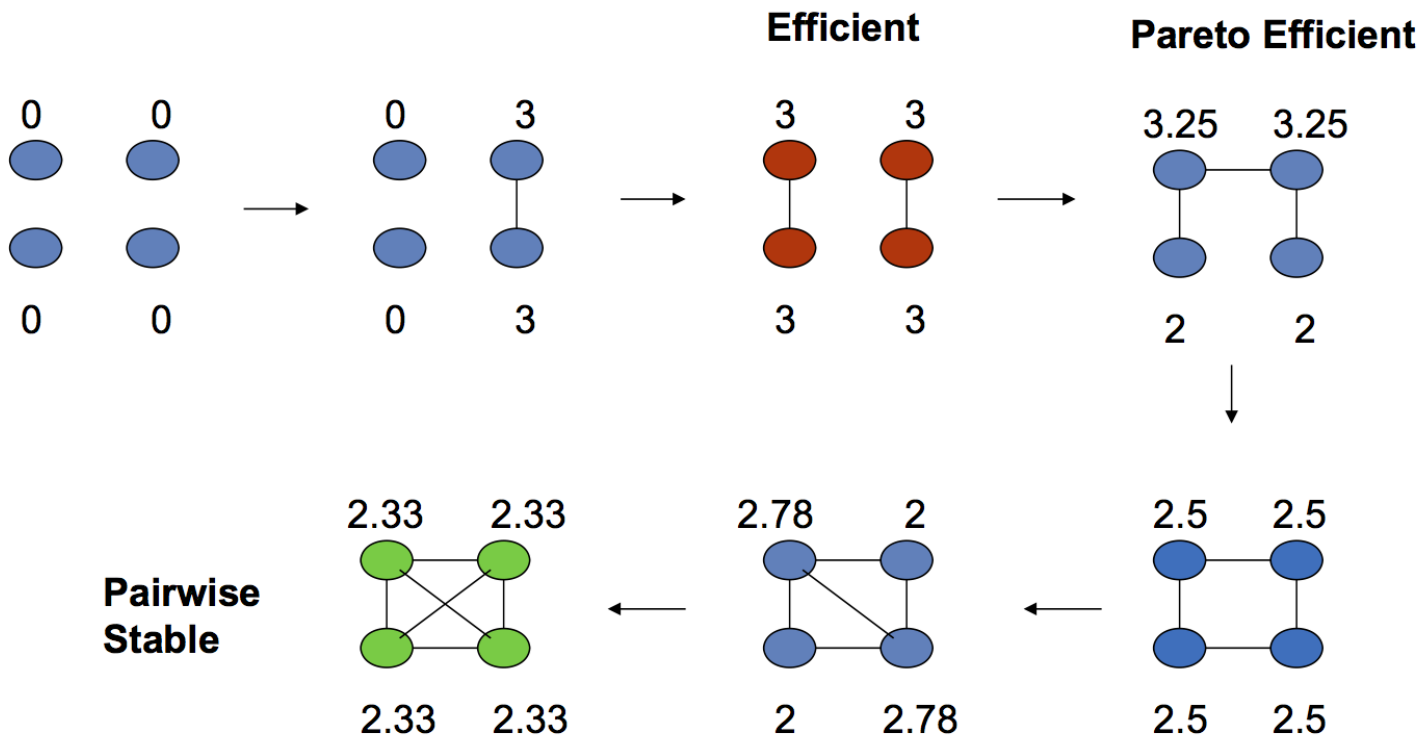
$$G^* = \max_G \sum_i u_i(G)$$

- Pareto Efficiency:

$$\begin{aligned} \nexists\, G' : \; u_i(G') &\geq u_i(G) \text{ for all } i \\ \text{and } u_i(G') &> u_i(G) \text{ for one } i \end{aligned}$$

- Efficiency $\Rightarrow$ Pareto Efficiency

Network Efficiency



References

- A Strategic Model of Social and Economic Networks, M. Jackson, A. Wolinksy, J. of Economic Theory, 71, pp44-74, 1996.